

## MATH 54 – MIDTERM 3

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Name: \_\_\_\_\_

**Instructions:** This midterm counts for 20% of your grade. You officially have 90 minutes to take this exam. May your luck be diagonalizable! :)

1		10
2		15
3		30
4		25
5		20
Bonus		2
Total		100

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*Date:* Friday, July 27th, 2012.

1. (10 points, 2 points each)

Label the following statements as **T** or **F**. **Write your answers in the box below!**

**NOTE:** In this question, you do **NOT** have to show your work! Don't spend *too* much time on each question!

- (a) If  $A$  is diagonalizable, then  $A^3$  is diagonalizable.
- (b) If  $A$  is a  $3 \times 3$  matrix with 3 (linearly independent) eigenvectors, then  $A$  is diagonalizable
- (c) If  $A$  is a  $3 \times 3$  matrix with eigenvalues  $\lambda = 1, 2, 3$ , then  $A$  is invertible
- (d) If  $A$  is a  $3 \times 3$  matrix with eigenvalues  $\lambda = 1, 2, 3$ , then  $A$  is (always) diagonalizable
- (e) If  $A$  is a  $3 \times 3$  matrix with eigenvalues  $\lambda = 1, 2, 2$ , then  $A$  is (always) not diagonalizable

(a)	
(b)	
(c)	
(d)	
(e)	

2. (15 points) Label the following statements as **TRUE** or **FALSE**. In this question, you **HAVE** to justify your answer!!!

This means:

- If the answer is **TRUE**, you have to explain **WHY** it is true (possibly by citing a theorem)
- If the answer is **FALSE**, you have to give a specific **COUNTEREXAMPLE**. You also have to explain why the counterexample is in fact a counterexample to the statement!

**IMPORTANT NOTE:** If  $A$  is diagonalizable, explain why! And if  $A$  is not diagonalizable, *show* why it isn't!

- (a) (5 points) If  $A$  is diagonalizable, then  $A$  is invertible.

(b) (*10 points, longer*) If  $A$  is invertible, then  $A$  is diagonalizable

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3. (30 points) Find a diagonal matrix  $D$  and an invertible matrix  $P$  such that  $A = PDP^{-1}$ , where:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

**Note:** Show *all* your work!

**Helps:** In case you're stuck:

- For 5 points, I can give you one root of the characteristic polynomial (ask me about it)
- For 10 points, I can give you the full characteristic polynomial! (ask me about it)

(Continuation)

4. (25 points) Solve the following system  $\mathbf{x}' = A\mathbf{x}$ , where:

$$A = \begin{bmatrix} 0 & 5 & 0 \\ -1 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

**Note:** Show *all* your work!



(Continuation)

5. (20 points, 10 points each)

Find the general solution to  $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ , where:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \mathbf{f}(t) = \begin{bmatrix} e^{4t} \\ e^{4t} \end{bmatrix}$$

**Note:** You may use the fact that the general solution to  $\mathbf{x}' = A\mathbf{x}$  is:  $\mathbf{x}_0(t) = Ae^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + Be^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) (10 points) Using undetermined coefficients

(b) (10 points) Using variation of parameters

**Note:** Simplify your answer!

**Bonus** (2 points)

(a) (0 points, but it'll help you for (b)) What is the general solution of  $y'' = -b^2y$

(b) (2 points) Use (a) and the *ideas* we talked about in lecture about the matrix exponential function to solve the following system  $\mathbf{x}'' = A\mathbf{x}$  (note the double prime), where:

$$A = \begin{bmatrix} 2 & -3 \\ 6 & 7 \end{bmatrix}$$

**Hint:** You may use the fact that  $A = -B^2$ , where  $B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$  as well as the fact that  $B = PDP^{-1}$ , where  $P = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ ,  
 $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

**Note:** Converting this into a first-order system differential equations is a complete waste of time! Do it directly using the hint above!

**Note:** This problem is a tiny bit long, but you really need to write down the final answer to get full credit!

(Continuation)

(Scratch work)

Any comments about this exam? (too long? too hard?)